	37	78	29	70	21	62	13	54	5	
46	6	38	79	30	71	22	63	14	46	6
	47	7	39	80	31	72	23	55	15	47
	16	48	8	40	81	32	64	24	56	16
	57	17	49	9	41	73	33	65	25	57
	26	58	18	50	1	42	74	34	66	26
	67	27	59	10	51	2	43	75	35	67
	36	68	19	60	11	52	3	44	76	36
	77	28	69	20	61	12	53	4	45	77
	37	78	29	70	21	62	13	54	5	46

AVERAGE AND PROBABILITY.

79. Proposed by the late ENOCH BEERY SEITZ.

Two equal spheres touch each other externally. If a point be taken at random within each sphere, show that (1) the chance that the distance between the points is less than the diameter of either sphere is 13/35, and (2) the average distance between them is $11/5r$. [This is problem 5835, *Educational Times*, of London.]

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

(1) Let A, B be the centers and C the point of contact of the two spheres, each radius r.

From any point P in DC with a radius$=2r$ describe a sphere cutting B in Q, R. From B as a center with a radius BP describe a sphere cutting A in K, M. If P is the first point, the second point must fall within the double-convex lens $CQRC$. P may fall anywhere on the zone KPM and the second point must fall in a section of B equal to the double-convex lens $CQRC$.

From P as center with a radius $PS<2r$ but$>PC$, draw the zone SLT. Let $DP=x$, $PS=y$, area of zone $KPM=2\pi.BP.PG$, area of zone $SLT=2\pi.PS.HL$.

$BP=3r-x$, $AG=r-x-PG$, $BG=3r-x-PG$, $PS=y$, $BH=3r-x-y+HL$, $PH=y-HL$.

$KG^2=r^2-(r-x-PG)^2=(3r-x)^2-(3r-x-PG)^2$.

$\therefore PG=x(2r-x)/4r$.

$SH^2=r^2-(3r-x-y+HL)^2=y^2-(y-HL)^2$.

$\therefore HL=[r^2-(3r-x-y)^2]/2(3r-x)$.

$\therefore$ Area of zone $KPM=(\pi x/2r)(3r-x)(2r-x)$.

Area of zone $SLT=[\pi y/(3r-x)][r^2-(3r-x-y)^2]$.

Let $p=$chance, $\triangle=$average distance.

$$\therefore\ p=\{\pi^2/[2r(\tfrac{4}{3}\pi r^3)^2]\}\int_0^{2r}x(3r-x)(2r-x)dx\int_{2r-x}^{2r}[y/(3r-x)][r^2-(3r-x-y)^2]dy$$

$$=(3/128r^7)\int_0^{2r}(14rx^5-x^6-48r^2x^4+48r^3x^3)dx=(3/128r^7)(1664r^7/105)=\tfrac{13}{35}.$$

$$2.\ \triangle=\{\pi^2/[2r(\tfrac{4}{3}\pi r^3)^2]\}\int_0^{2r}x(3r-x)(2r-x)dx\int_{2r-x}^{4r-x}[y^2/(3r-x)][r^2-(3r-x-y)^2]dy$$

$$=(3/40r^4)\int_0^{2r}(92r^3x-106r^2x^2+40rx^3-5x^4)dx=(3/40r^4)(88r^5/3)=11r/5.$$

80. **Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.**

A box contains 100 balls marked from 1 to 100. 13 balls are drawn at random. What is the chance that the balls marked from 1 to 10 are included in the 13 drawn ?

Solution by J. W. YOUNG, Columbus, Ohio.

Since in all the favorable chances only three balls may vary, the total number of favorable chances is ${}^{90}C_3$, *i. e.*, the number of combinations of 90 things taken 3 at a time.

The total number of ways in which the balls may be drawn is, of course, ${}^{100}C_{13}$.

Hence the desired probability is equal to

$$\frac{{}^{90}C_3}{{}^{100}C_{13}}=\frac{\dfrac{90.89.88}{1.2.3}}{\dfrac{100.99.98.97.96.\ldots.89.88}{1.2.3.5\ldots\ldots13}}=\frac{1}{67515927540}.$$

PROBLEMS FOR SOLUTION.

ARITHMETIC.

124. **Proposed by ALOIS F. KOVARIK, Instructor in Mathematics and Science, Decorah Institute, Decorah, Iowa.**

At what time between 5 and 6 o'clock is the minute hand midway between 12 and the hour hand ? When is the hour hand midway between 4 and the minute hand ?